A History of Compass and Triangle

March 3, 1999

*We are familiar with the compass and triangle but few people are aware of its history. I would like to show you its history in this report, which is an excerpt from my last lecture notes at the Asakita senior high school that I retired from last year. Because this is not a full study, I am afraid this may have a lot of mistakes, if you would point out my errors in my report, I appreciate it. I affixed the reference marks *) to the upper right of quotations and put them at the bottom of the illustrations to trace to their original sources.*

1. Introduction

Convention, rather than utility, dictates that we are to use the compass and the straightedge, mathematical implements which date from Classical Greek civilization, to draw figures in geometry according to Euclidian postulates, though the both tools lack graduations. With the compass and the straightedge we can:

1) *Draw a line which passes through two arbitrary points.*
2) *Draw a circle with a center at an arbitrary point with a given radius.*

Under such inconvenient restrictions, it took many years of research to solve a number of difficult mathematical problems including:

1) *Quadrature of circle*
2) *Trisection of angle*
3) *Duplication of cube*

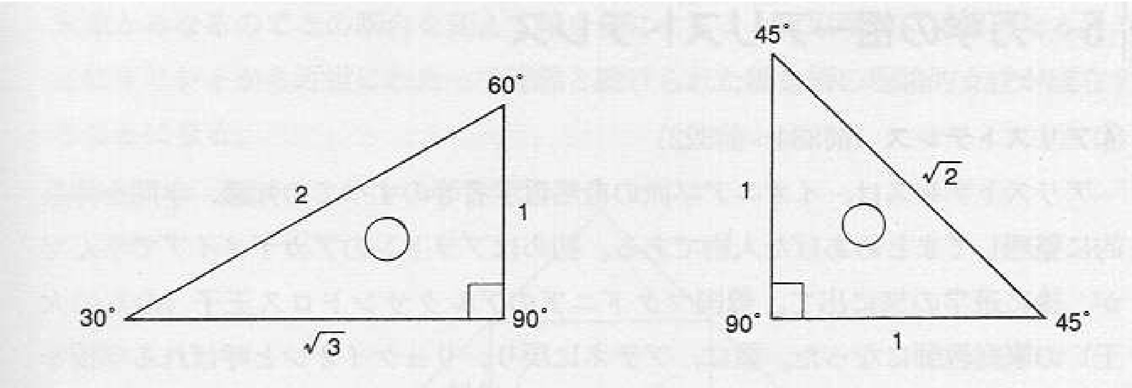
In the traditional Japanese teaching of mathematics, the same strictures held true. Nowadays, however, we do not adhere so closely to the restrictions. According to the teaching guidelines of the Ministry of Education, the following methods of instruction regarding the compass and the triangle are recommended:

・*In elementary school, the implements are used to draw triangles and quadrangles in the third grade and to draw polygons by drawing circles in the fifth grade.* [13)]

・*In junior high school, the basic construction, such as drawing perpendicular bisectors, angular bisectors and perpendiculars, recommended in the unit "figure."* [14)]

・*In senior high school, we have the unit "plane geometry," but the construction is not recommended at all.* [15)] *In the unit "trigonometric function," the triangle is only utilized as follows:*

Find the trigonometric ratios, sin45°, cos30°, and tan60° in the following triangles.

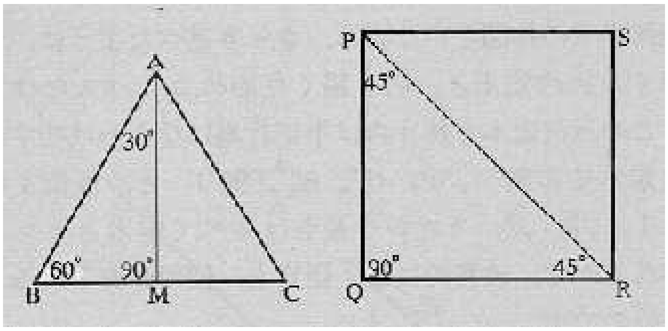


(Fig. 1)

2. Shape of the triangle

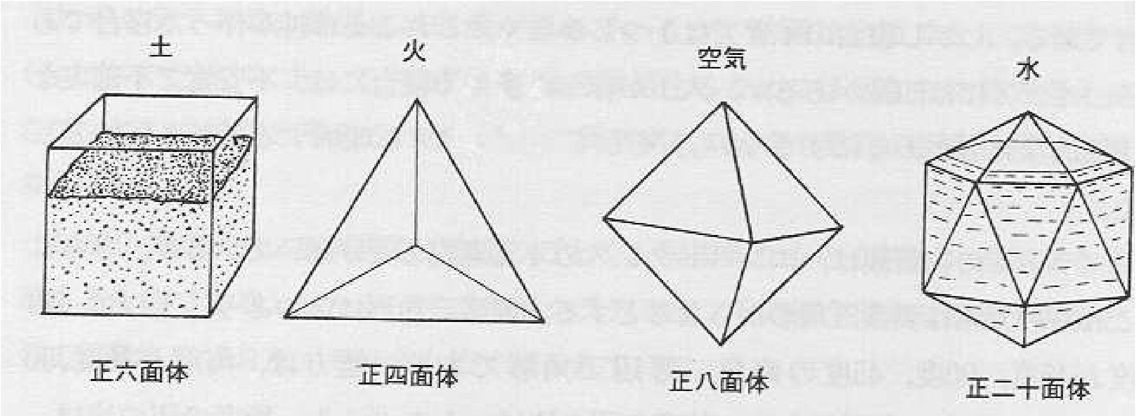
This figure shows pair of triangle consisting of two right triangles. (Fig. 1) Why do they take such a shape and have a round hole at the center? Let's try to clarify the reasons.

(1) The regular triangle and the square, perhaps the most beautiful figures in geometry, and the right angle is frequently used in construction problems. Therefore, as seen in Figure 2, we see the right triangle bisecting the regular triangle with an axis through a vertex, and bisecting a square with a diagonal. Thus, these right triangles are the original form of the triangle.[6] *(Fig. 2)*



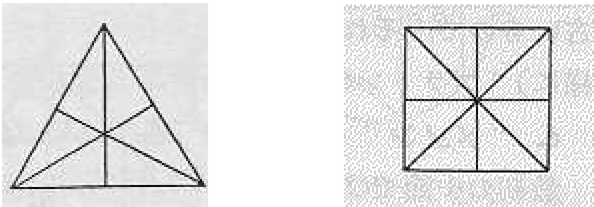
(Fig. 2)[6]

(2) The Greek philosopher Plato said, in his theory of ideas, that all things under the sun were made from four elements — fire, air, water and the earth. These four elements were made, in turn, made from the regular tetrahedron, the regular octahedron, the regular icosahedron and the cube, respectively. (Plato's figures)[4,5] *(Fig.3)*



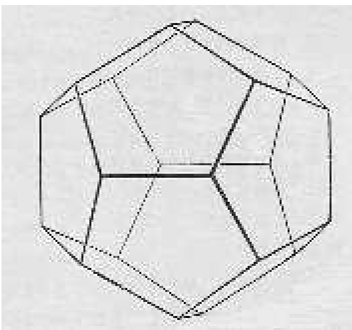
(Fig. 3) [5]

The regular polyhedron consists of regular triangles and squares. By bisecting these two figures with a perpendicular line and a diagonal line, Plato found two right triangles (elementary triangles) [2, 5] *(στοικεια)* [18] *(Fig. 4)*

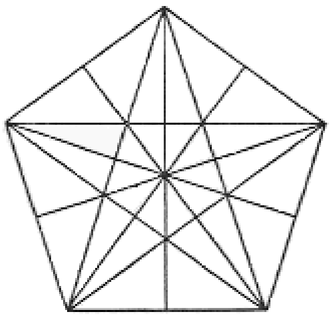


(Fig. 4) [2]

Furthermore, in a dodecahedron, the outlines of all the faces are regular pentagons, which he bisected with perpendiculars and diagonals into thirty right triangles. He made a correspondence between the Creator (of the Universe) and the dodecahedron, which differentiated from the elementary triangles.



(Fig. 5) [4]



(Fig. 6) [2]

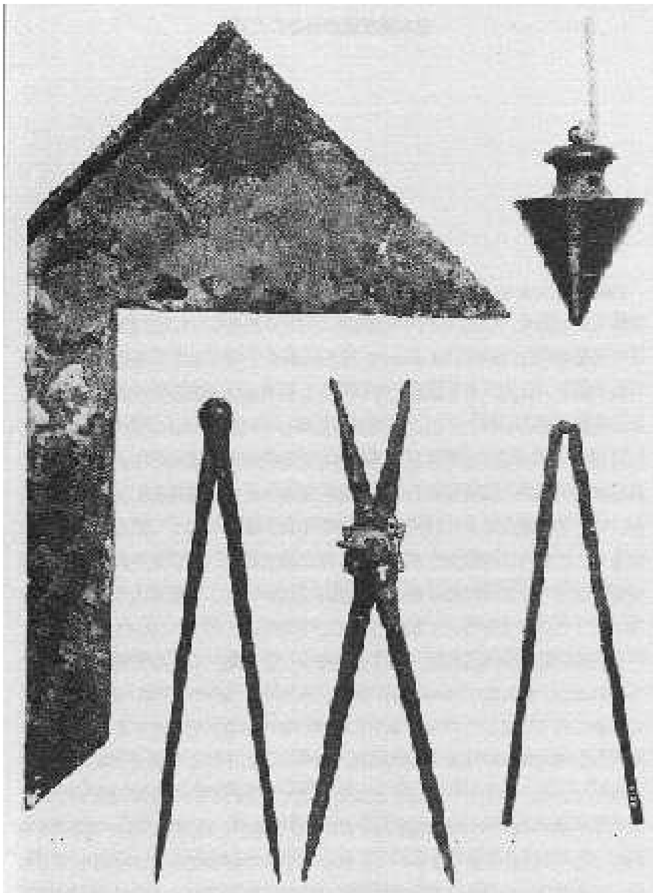
It is my belief that Plato's elementary triangles themselves did not determine the

shape of a triangle, but rather, they were adopted as basic figures in drafting.

(3) The triangle is ordinarily made of cherry, Japanese cypress (hinoki), zelkova, bamboo, and plastics but highly valued ones are made of pear. The triangle is mainly made of wood because plastics are transparent but easily warped. In order to avoid warping, three sheets of wood are put together to create a triangle. Due to the result, the triangle has a triangular hole at the center — the round hole is a vestige of the triangular hole which remains since it is convenient for us to shift the triangle by the hole. [17]

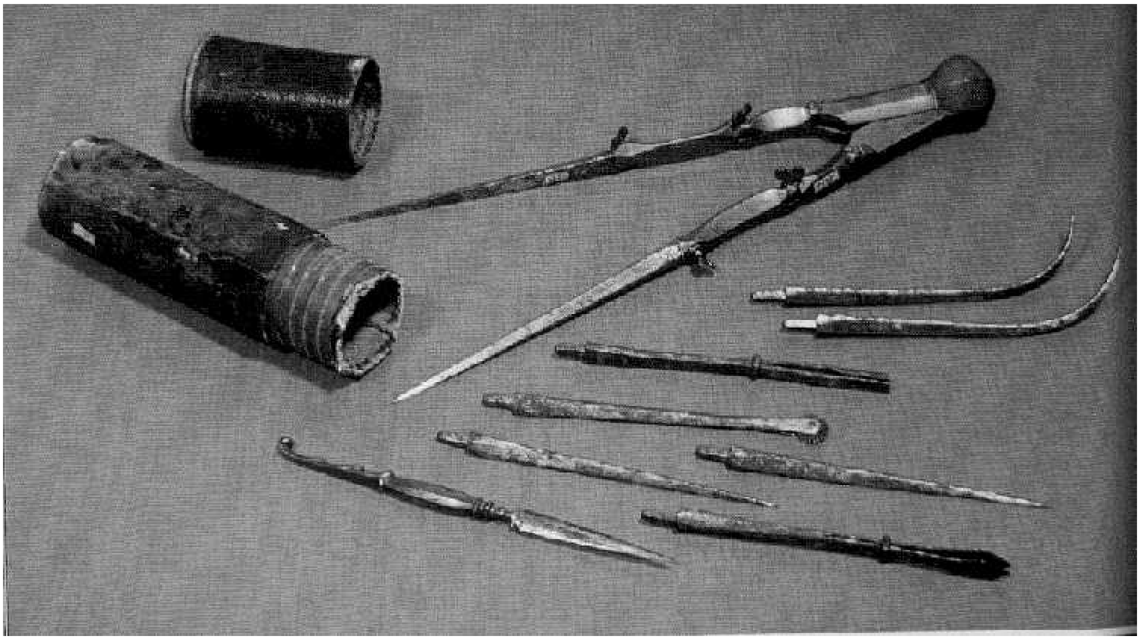
3. Advent of compass and triangle

The ruler and the compass were used by the ancient Greeks as basic teaching tools, and there seemed to be used the double divisible compass. The protractor had not yet been discovered, though the Greeks utilized some form of fixed squares to measure right angles. [3] *The figure below (Fig. 7) shows a triangle, a divisible compass, a proportional compass, a ruler and a weight in Roman times. (From the British Museum's collection)* [3]
These can easily be seen in comparison to contemporary drafting tools.



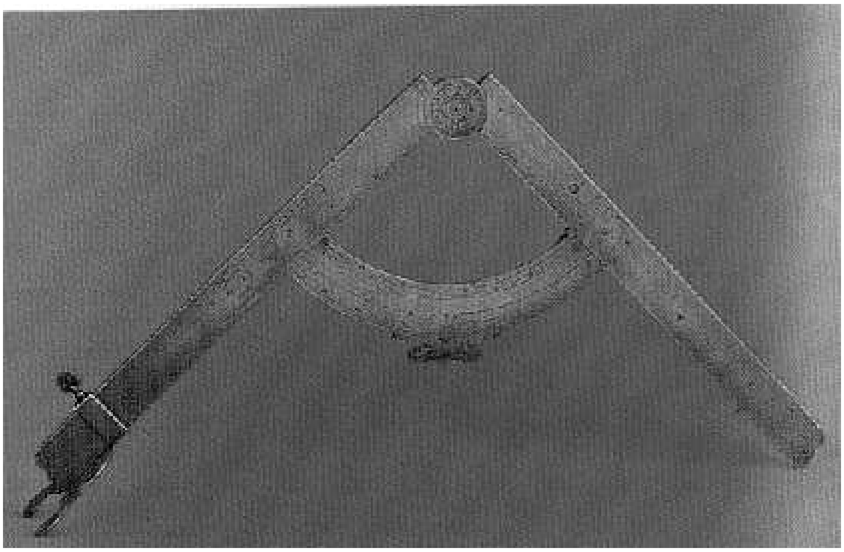
(Fig.7) [3]

During the Dark Ages (which preceded the European Renaissance), the discipline of mathematics in Europe fell into its dormancy. In subsequent centuries, the Greek study of mathematics traveled to India and in the 7th century, was developed in Arabia before returning to Europe in the 12th century. At that time, "Aldschebr walmukabala," a study of algebra authored by Al-Khwarizmi, became popular in Europe. Indeed, the word algebra is derived from the first part of the book's title Aldschebr, while the author's name gave us the term algorism. Importantly, the triangle remained essentially unchanged through the time of the book's popularity. The figure below (Fig. 8) shows a compass purported to be the property of Michelangero; it is made of brass and its edge is composed of steel. As one can see, the form of the compass had not changed through the 16th century.

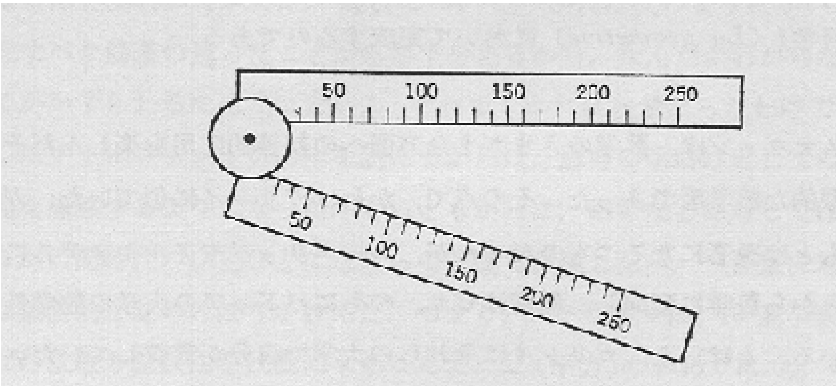


(Fig. 8) [12)]

In 1606, Galileo Galilei published his thesis, "Le operazioni del compasso geometrico et militare," (The operation of geometric and military compass). Galileo graduated the compass, and was able to demonstrate the calculation of compound interest and reduction of drafting. [16)] *The following photograph (Fig. 9) shows a 25.5 centimeter compass which Galileo devised and dedicated to Cosimo de Mediti (1590 — 1621). Figure 10 shows its graduations.*



(Fig. 9) [12)]



(Fig. 10)[16)]

4. Introduction to Japan

The compass and drafting were brought to Japan by the Portuguese. According to surveyors in the Edo period, Dutchman Caspar taught graphics to Kanesada Higuchi during the "Kan-ei" era (1624— 1643). [9)] *However, this may be an expedient notion which serves to hide the introduction to Japan of these implements during the period of national seclusion. The compass was mainly used as a divider but sometimes it was utilized with a writing implement for drafting purposes. It had been, at one time, called "bunmawashi," but it was referred to as "enki"* 円規 and *"konpasu"* 渾発（根発） *after the time of its introduction.*

"Konpasu" has as its origin the Portuguese "compasso." The Japanese word "konpasu" 根発子 *appears in the illustrated reference book "wakan-sansai-zue"* 和漢三才図会. *(1715)* [11)] *There is another account which holds that a French man brought the compass to Japan*

and that "konpasu" is derived from the French word "compass." ("kiku-genpo-choken -bengi")

The compass and the triangle are a part of the draftsman's toolbox. It is said that even the cartographer Tadataka Ino and the explorer Rinzo Mamiya fashioned their own compasses of bamboo. Teiichiro Wada was taught graphics and casting by an English man, and armed with knowledge of both disciplines, forged drafting implements in 1869, thus, he produced the first modern compass and triangle in Japan. [10)]

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